# THREE PHASE A.C CIRCUIT

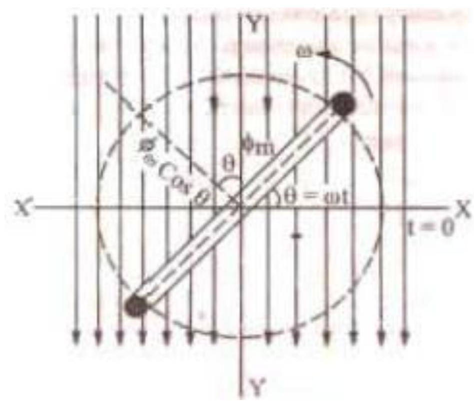
## Single phase EMF generation:

Alternating voltage may be generated

1) By rotating a coil in a magnetic field
2) By rotating a magnetic field within a stationary coil

The value of voltage generated depends upon
1) No. of turns in the coil 2) field strength 3) speed

## Equation of alternating voltage and current




N= No. of turns in a coil

$\Phi_m$= Maximum flux when coil coincides with X-axis

$\omega$= angular speed (rad/sec) = $2\pi f$

At $\theta=\omega t$, $\Phi$= flux component $\perp$ to the plane $=\Phi_m \cos \omega t$

According to the Faraday's law of electromagnetic induction,

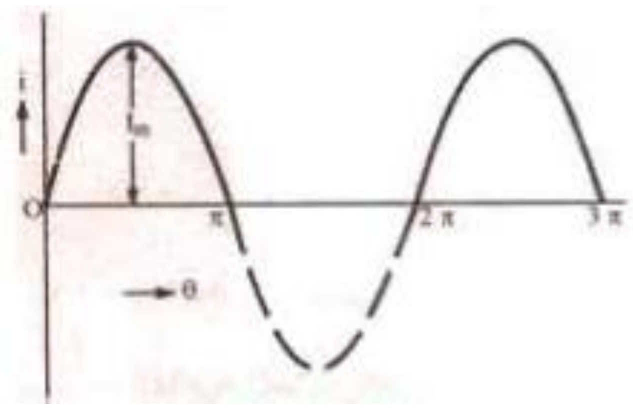
$$e=-N\frac{d\varphi}{dt}=-N\frac{d}{dt}\varphi_m \cos\omega t=\omega N\varphi_m \sin\omega t \text{...........}(1)$$

Now, e is maximum value of $E_m$ , when $\sin\theta = \sin 90° = 1$.

i.e $E_m = \omega N\Phi_m$ ……………………………………………...(2)

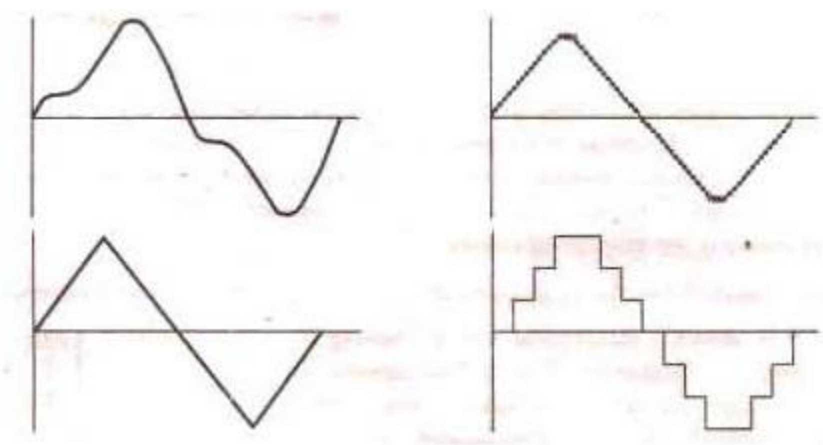
From $Eq^n$ (1) & (2), $e = E_m \sin \omega t$ volt

Now, current (i) at any time in the coil is proportional to the induced emf (e) in the coil. Hence, $i = I_m \sin \omega t$ amp



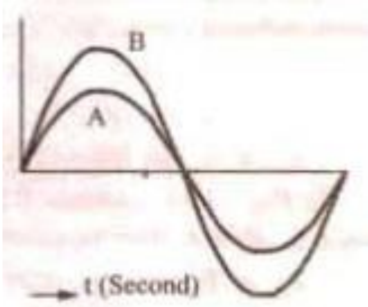

<u>A.C</u> **<u>terms</u>**:

- Cycle:- A complete set of positive and negative values of an alternating quantity is known as cycle.



- Time period: The time taken by an alternating quantity to complete one cycle is called time T.
- Frequency: It is the number of cycles that occur in one second. $f = 1/T$
  $f = PN/120$ where, P= No. of poles, N= Speed in rpm
- Waveform: A curve which shows the variation of voltage and current w.r.t time or rotation.

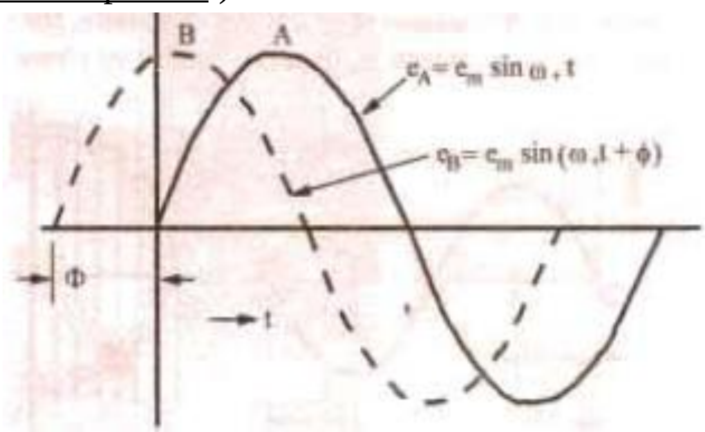
- Phase & Phase difference:




In phase: $e_A = E_{mA} \operatorname{Sin} \omega t$

$e_B = E_{mB} \operatorname{Sin} \omega t$

Out of phase: i) B leads A




Phase difference Φ. $e_A = E_m \operatorname{Sin} \omega t$

$e_B = E_{mB} \operatorname{Sin} (\omega t + \alpha)$

ii) A leads B or B lags A

$e_A = E_m \operatorname{Sin} \omega t$

$e_B = E_m \operatorname{Sin} (\omega t - \alpha)$

**Example**: An alternating current varying sinusoidally with a frequency of 50 Hz has an RMS value of 20 A. Write down the equation for the instantaneous value and find this value a) 0.0025 sec b) 0.0125 sec after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A?

Ans: $I_m = 20\sqrt{2} = 28.2\,A$

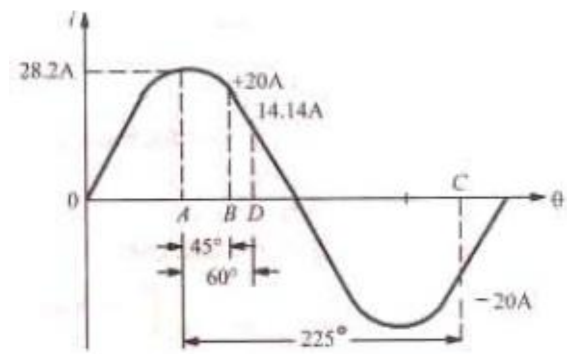
$\omega = 2\pi \times 50 = 100\pi\ rad/s$

The equation of the sinusoidal current wave with reference to point O as zero time point is

i= 28.2 sin 100πt Ampere

Since time values are given from point A where voltage has positive and maximum value, the equation may itself be referred to point A. In this case, equation becomes

i= 28.2 cos 100πt




i) When t= 0.0025 second

i = 28.2 cos 100π X 0.0025 ………….angle in radian

= 28.2 cos 100 X 180 X 0.0025 ………….angle in degrees

= 28.2 cos 45° = 20 A ……………………point B

ii) When t = 0.0125 sec

I =28.2 cos 100 X 180 X 0.0125

= 28.2 cos 225° = 28.2 X (-1/√2)

= -20 A …………………………point C

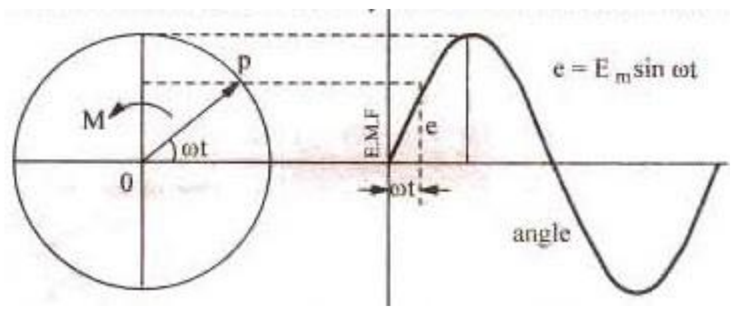
iii) Here i = 14.14 A

14.14 = 28.2 COS 100 X 180 t

Cos 100 X 180 t = ½

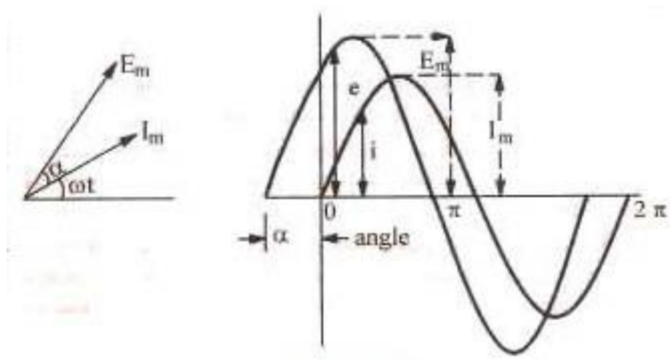
Or, 100 X 180 t = $\cos^{-1}(1/2)$ = 60°, t=1/300 sec …………..point D

## <u>Phasor & Phasor diagram:</u>




**<u>Phasor</u>**: Alternating quantities are vector (i.e having both magnitude and direction). Their instantaneous values are continuously changing so that they are represented by a rotating vector (or phasor). A phasor is a vector rotating at a constant angular velocity

**<u>Phasor diagram</u>**: is one in which different alternating quantities of the same frequency are represented by phasors with their correct phase relationship




**Points to remember:**

1. The angle between two phasors is the phase difference
2. Reference phasor is drawn horizontally
3. Phasors are drawn to represent rms values
4. Phasors are assumed to rotate in anticlockwise direction
5. Phasor diagram represents a "still position" of the phasors in one particular point

**Power Factor**

The phase angle of the load impedance plays a very important role in the absorption of power by load impedance. The average power dissipated by an AC load is dependent on the cosine of the angle of the impedance. To recognize the importance of this factor in AC power computations, the term cos(θ ) is referred to as the power factor (pf). Note that the power factor is equal to 0 for a purely inductive or capacitive load and equal to 1 for a purely resistive load; in every other case,0<pf<1.If the load has an inductive reactance, then θ is positive and the current lags (or follows) the voltage. Thus, when θ and Q are positive, the corresponding power factor is termed lagging. Conversely, a capacitive load will have a negative Q, and hence a negative θ. This corresponds to a leading power factor, meaning that the load current leads the load voltage.A power factor close to unity signifies an efficient transfer of energy from the AC source to the load, while a small power factor corresponds to inefficient use of energy .Two equivalent expressions for the power factor are given in the following:

$$\text{pf} = \cos(\theta) = \frac{P_{\text{av}}}{\tilde{V}\tilde{I}} \qquad \text{Power factor}$$

where $\tilde{V}$ and $\tilde{I}$ are the rms values of the load voltage and current.

**Active, Reactive and Apparent Power**

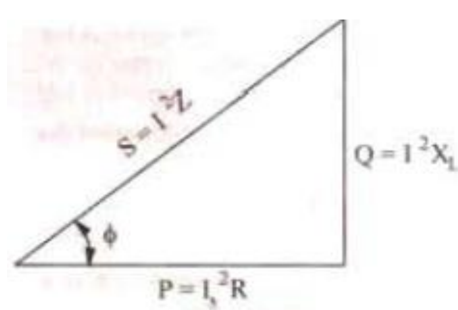
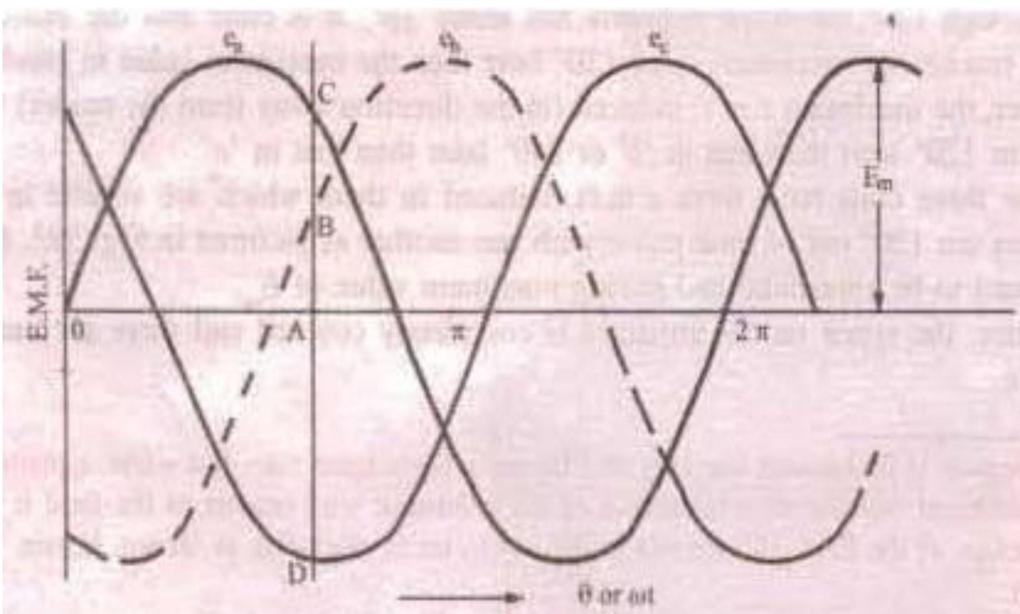



Fig. Power Triangle

$$S^2 = P^2 + Q^2$$

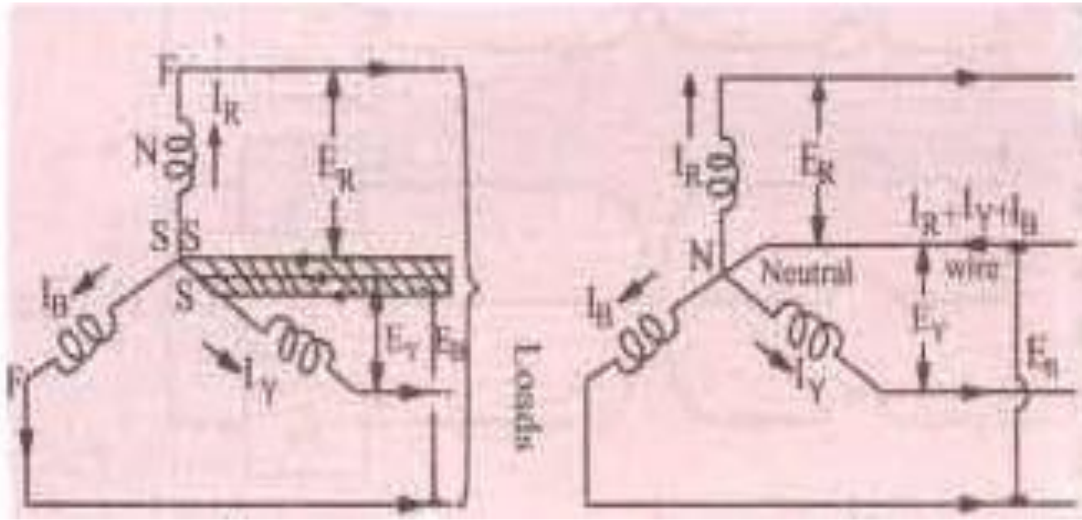
- **Apparent power, S**: is the product of rms values of the applied voltage and circuit current. It is also known as wattless (idle) component
  $S=VI=IZ\times I=I^2Z$ volt-amp
- **Active power or true power, P**: is the power which actually dissipated in the circuit resistance. It is also known as wattful component of power.
  $P=I^2R=I^2Z\cos\Phi=VI\cos\Phi$ watt
- **Reactive power, Q**:- is the power developed in the reactance of the circuit.
  $Q=I^2X=I^2Z\sin\Phi=VI\sin\Phi$ VAR

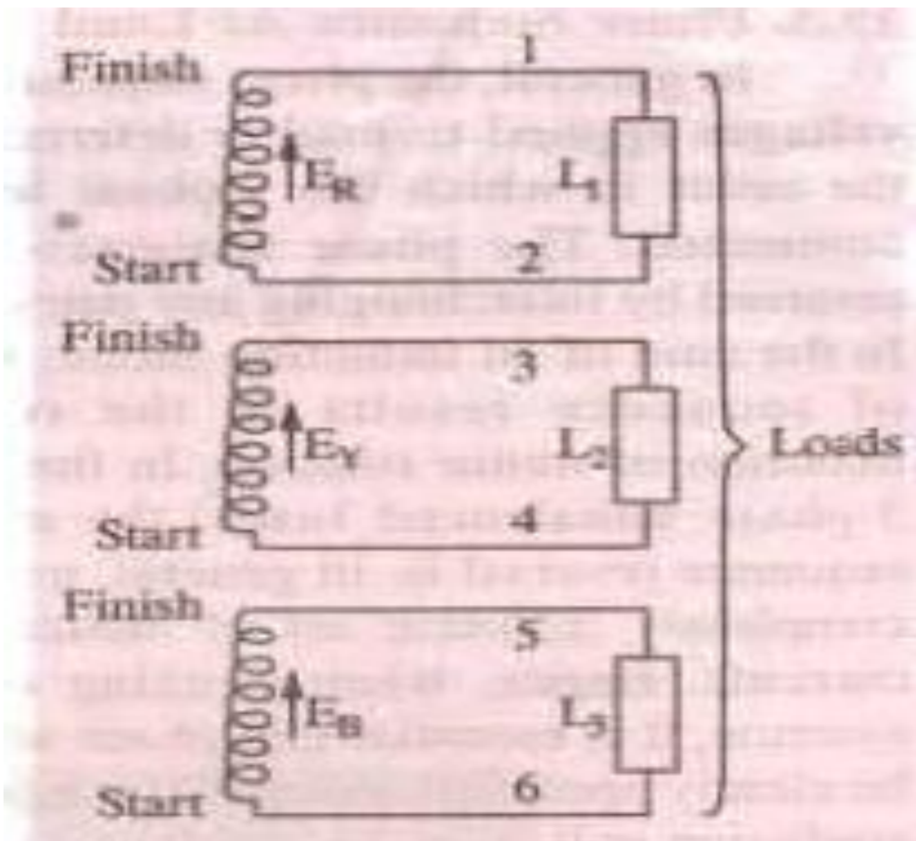
**Three phase EMF Generation**:-

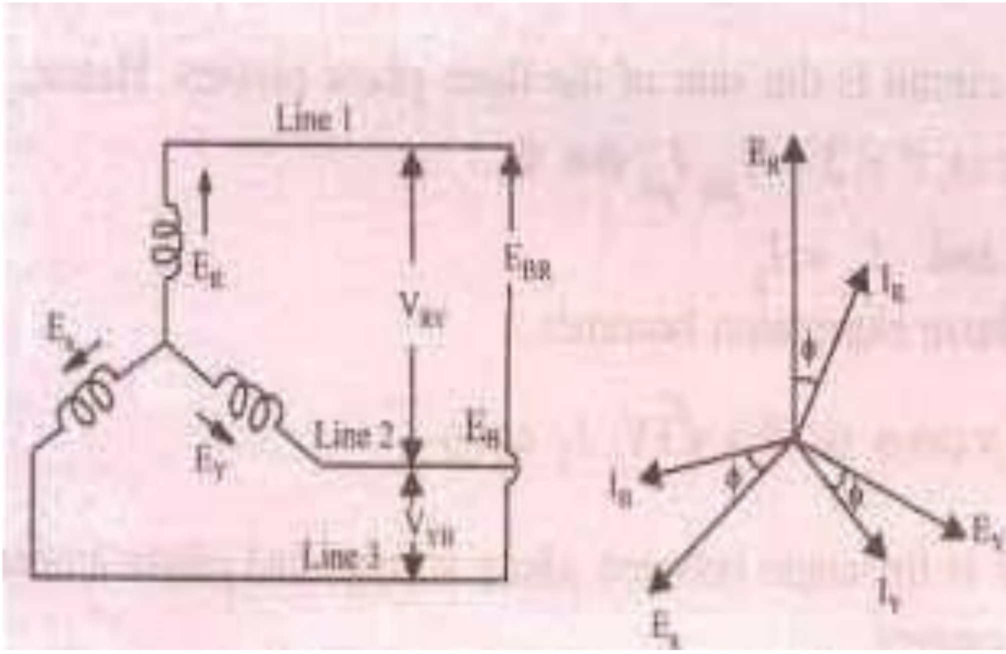



If the 3-coil windings $W_1$, $W_2$ and $W_3$ arranged at 120° apart from each other on the same axis are rotated, then the emf induced in each of them will have a phase difference of 120°. In other words if the emf (or current) in one winding ($w_1$) has a phase of 0°, then the second winding ($w_2$) has a phase of 120° and the third ($w_3$) has a phase of 240°.

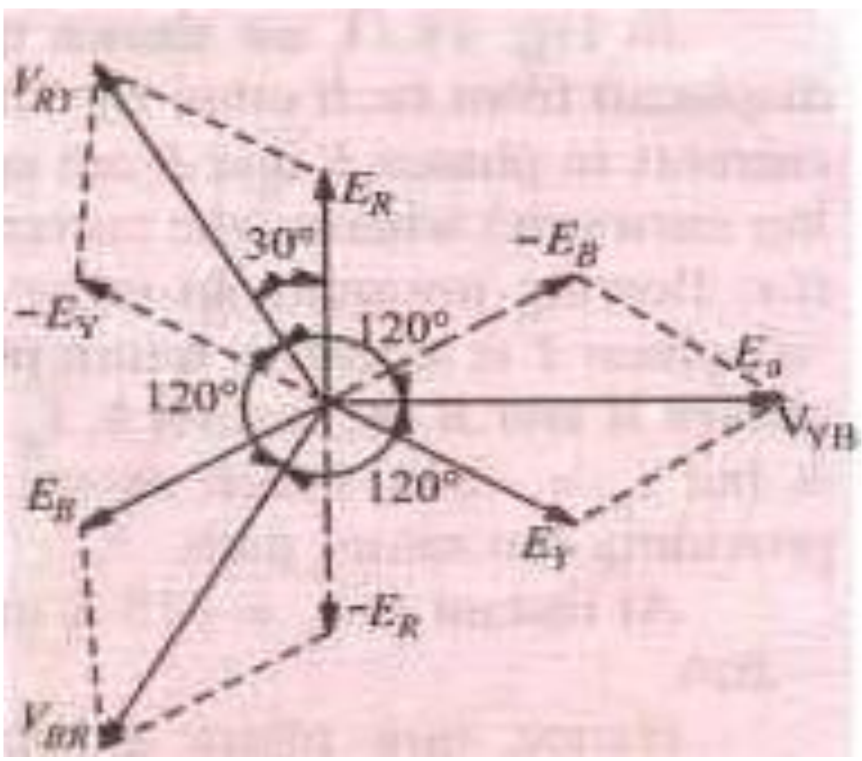
**Star (Y) connection**:-






Phasor diagram:-




Here, $E_R$, $E_Y$, $E_B$ are phase voltages and $V_{RY}$, $V_{YB}$, $V_{BR}$ are line voltages

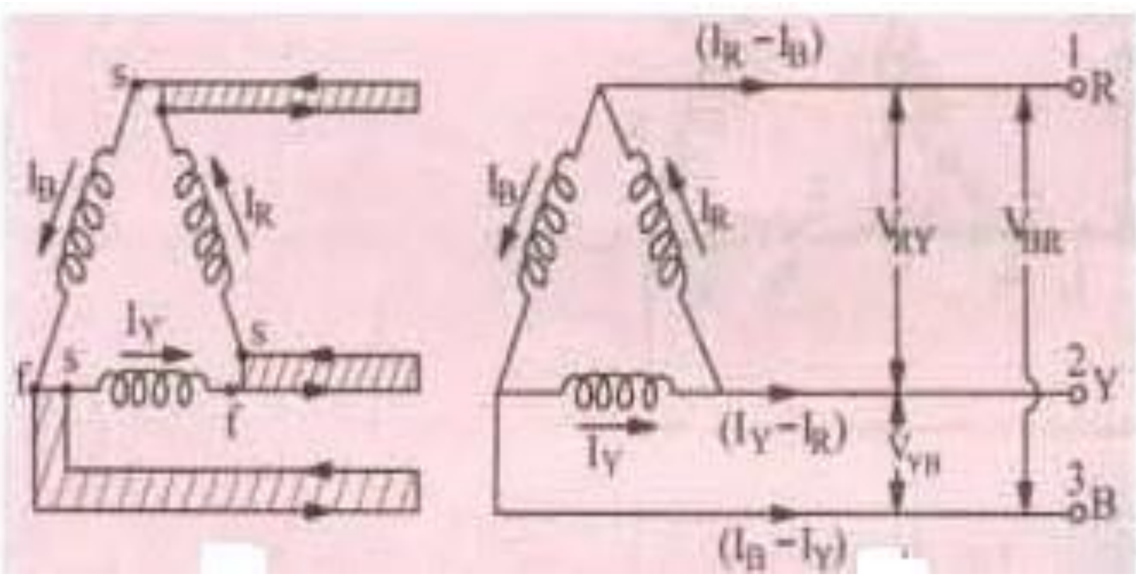
$$V_{RY} = \sqrt{E_R{}^2 + E_Y{}^2 + 2E_R E_Y \cos 60^o}$$
$$= \sqrt{E_R{}^2 + E_R{}^2 + 2E_R E_R \cos 60^o}$$
$$= \sqrt{3}\, E_R$$

Hence,

- Line voltage =√3 x phase voltage
- Line current = phase current
- Line voltages are also 120° apart
- Line voltage are 30° ahead of respective phase voltages
- The angle between line voltage and line current is (30°+Φ)

Power: Total power = 3 x phase power
=3 x $V_{ph}$ x $I_{ph}$ x cosΦ
=√3 $V_L$ $I_L$ cosΦ
Φ is the angle between phase voltage and current

**Delta-connection**:



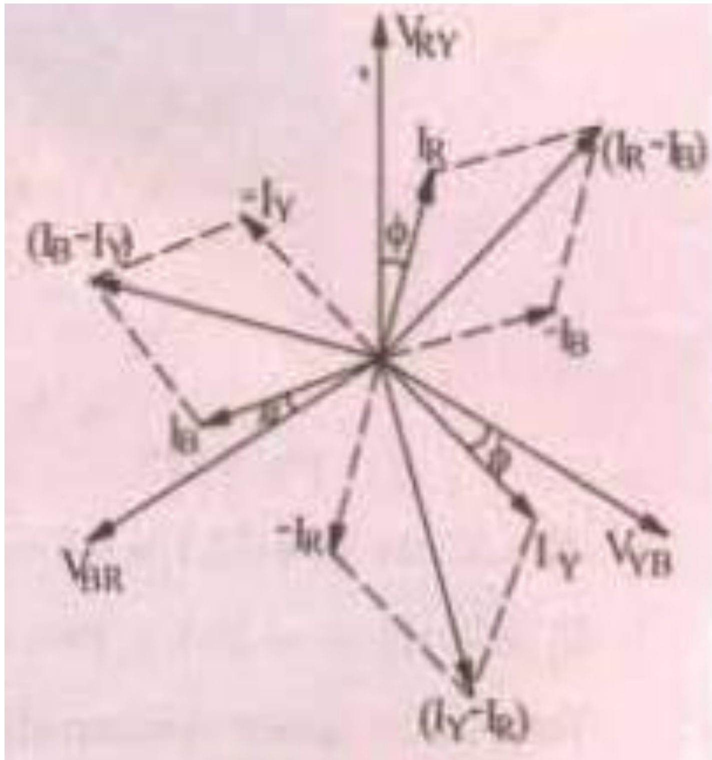



Fig. Phasor Diagram

$$I_L = I_R - I_B$$

$$I_L = \sqrt{I_R^2 + I_B^2 + 2 I_R I_B \cos 60^o} = \sqrt{I_R^2 + I_R^2 + 2 I_R I_R \cos 60^o} = \sqrt{3} I_R$$

Hence,

- Line current =√3 phase current
- Line voltage = phase voltage
- Line currents are also 120° apart
- Line currents are 30° behind the respective phase currents
- Angle between line current and line voltage is 30°+Φ

Power: Total power = 3 x phase power

= 3 x $V_{ph}$ $I_{ph}$ cosΦ

= 3 x $V_L$ x $I_L$/√3 x cosΦ

= √3 $V_L$ $I_L$ cosΦ

**Note**: For both star and delta system:

Active & True power = $\sqrt{3}\ V_L\ I_L \cos\Phi$

Reactive power =$\sqrt{3}\ V_L\ I_L \sin\Phi$

Apparent power =$\sqrt{3}\ V_L\ I_L$

## STAR-DELTA CONVERSION

**Need**:- Complicated networks can be simplified by successively replacing delta mesh to star equivalent system and vice-versa.

In delta network, three resistors are connected in delta fashion (Δ) and in star network three resistors are connected in wye (Y) fashion.

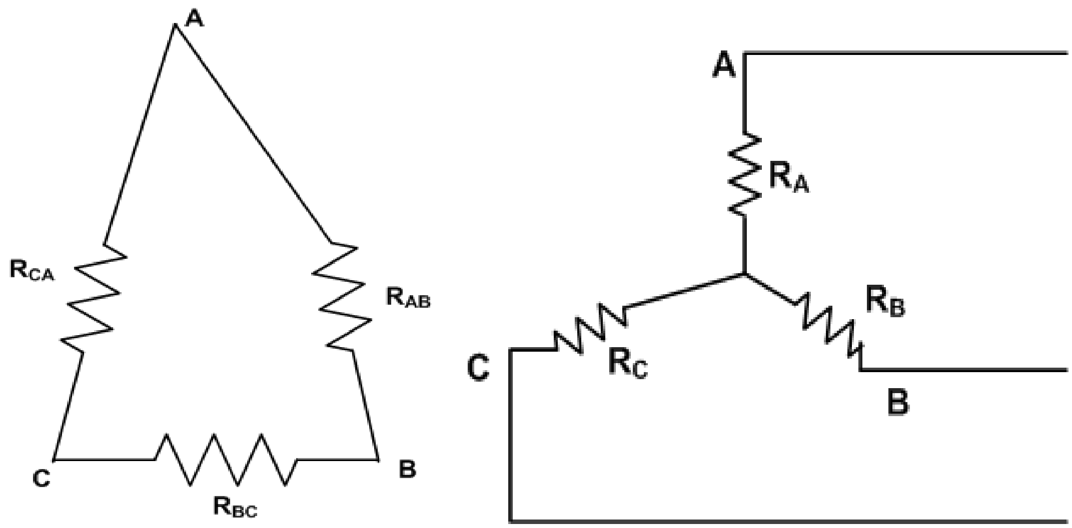



Fig. 1.4.1.

a) Delta connection b) Star connection

**Delta to Star Conversion**:- From Fig. 1.4.1 (a), Δ : Between A & B, there are two parallel path.

Resistance between terminal A & B = $\dfrac{R_{AB}(R_{BC}+R_{CA})}{R_{AB}+R_{BC}+R_{CA}}$

From Fig. 1.4.1 (b), STAR: Between A & B two series resistances are there $R_A + R_B$. So, terminal resistances have to be the same.

$$R_A + R_B = \frac{R_{AB}(R_{BC}+R_{CA})}{R_{AB}+R_{BC}+R_{CA}} \;\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$R_B + R_C = \frac{R_{BC}(R_{CA}+R_{AB})}{R_{AB}+R_{BC}+R_{CA}} \;\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$R_C + R_A = \frac{R_{CA}(R_{AB}+R_{BC})}{R_{AB}+R_{BC}+R_{CA}} \;\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Eq {(1)-(2)}+(3) & Solving,-

$$R_A = \frac{R_{AB}\times R_{CA}}{R_{AB}+R_{BC}+R_{CA}} \;\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$R_B = \frac{R_{AB}\times R_{BC}}{R_{AB}+R_{BC}+R_{CA}} \;\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$R_C = \frac{R_{CA} \times R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

**Easy way to remember**:-

$$\textit{Any arm of star connection} = \frac{\textit{Product of two adjacent arms of delta}}{\textit{sum of arms of delta}}$$

**<u>Star to Delta conversion</u>**

Eq {(1) X (2)}+(2) X (3)+ (3) X (1) & Simplifying,-

$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

**Easy way to remember**:- *Resistance between two terminals of delta = sum of star resistance connected to those terminals + product of the same to resistance divided by the third resistance.*